

INDIAN MARITIME UNIVERSITY
(Central University, Government of India)

May/June 2016 End Semester Examinations
B.Tech. (Marine Engineering)

Second Semester – Mathematics – II - (UG11T2202/UG11T1202)

Date : 10.06.2016

Time: 3 Hrs

Max Marks: 100

Pass Marks: 50

Part-A
Compulsory Question

(3 x 10 = 30 Marks)

- 1) a) Solve the differential equation $(x^2 - yx^2) \frac{dy}{dx} + (y^2 + xy^2) = 0$
b) Check the exactness of the equation $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$ and solve the equation.
c) A discrete random variable X has the probability density function $p(x) = 2^{-x}$, $x = 1, 2, 3, \dots$ calculate its moment generating function.
d) Find a_0 for the Fourier series expansion of the following function

$$f(x) = x + x^2 \text{ for } -\pi < x < \pi.$$

- e) Find the value of k when the function

$$f(x) = \begin{cases} kx, 0 \leq x < 2 \\ 2k, 2 \leq x < 4 \\ -kx + 6k, 4 \leq x < 6 \end{cases}$$

f)

Evaluate the integral $\int_0^a te^{-2t} \sin(3t) dt$ using Laplace transform method.

g)

If $F(s) = \log \frac{s^2 + 1}{s(s + 1)}$, find corresponding $f(t)$.

- h) Find the inverse Laplace transform of $\frac{3(s^2 - 2)^2}{2s^5}$

i) Solve $\frac{d^2y}{dx^2} + \frac{5dy}{dx} + 6x = 0$ given $y(0) = 0$ $\frac{dy}{dx}(0) = 15$

j) If on an average 1 vessel in every 10 is wrecked, find the probability that out of 5 vessels expected to arrive, atleast 4 will arrive safely.

Part-B

5 x 14 = 70 Marks)

Answer any five of the following

- 2) (a) Find the cosine series for the function f defined by

$$f(x) = \begin{cases} x, 0 \leq x \leq L/2 \\ L - x, L/2 \leq x \leq L \end{cases}$$

(b) Find the Laplace transform form of $f(t) = te^{3t} \sin t$ (7+7)

3) Solve the following differential equations :

(i) $3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$ (ii) $(3x^2 + 6xy^2) dx + (6x^2 y + 4y^3) dy = 0$

(iii) $(x+1) \frac{dy}{dx} - y = e^{3x} (x+1)^2$ (iv) $x^2 \frac{du}{dx} - u = 2 \sin(1/x)$ (7+7)

4) (a) In a bolt factory, machines A, B and C manufactured 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found defective. What are the probabilities that it was manufactured by the machines A, B or C ?

(b) A continuous probability distribution function is defined as

$$f(x) = ax(2-x), 0 \leq x \leq 2$$

Calculate the values of a , the mean, mode and the median of the distribution. (7+7)

5) Derive the Fourier series for the half-wave rectified sinusoidal wave defined by

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

and $f(t) = f(t + 2\pi)$ and hence deduce that

(a) $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots = \frac{1}{2}$

(b) $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi - 2}{4}$ (7+7)

6)

(a) Find the complete solution (C.S) for the non homogeneous differential

equation $\frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 6y = e^{-2x} \sin 2x$.

(b) Use method of variation of parameters to solve the following differential

equation $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$. (7+7)

7) a. A sortie of 20 aeroplanes is sent on an operational flight. The chances that an aeroplane fails to return is 5%. Find the probability that (i) one plane does not return. (ii) at the most 5 planes do not return.

b. A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days (i) on which there is no demand, (ii) on which demand is refused. ($e^{-1.5} = 0.2231$) (7+7)

- 8) a. A random variable gives measurements x between 0 and 1 with a probability function.

$$f(x) = \begin{cases} 12x^3 - 21x^2 + 10x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- i) Find $P\left(x \leq \frac{1}{2}\right)$ and $P\left(x > \frac{1}{2}\right)$
ii) Find a number k such that $P(x \leq k) = \frac{1}{2}$

- b. The probability density $p(x)$ of a continuous random variable is given by

$p(x) = y_0 e^{-|x|} - \infty < x < \infty$ Calculate y_0 , Mean and variance of the distribution. (7+7)
